Gauge Theory for Quantum Technologists

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From a certain perspective, we might consider quantum mechanics --- with respect to classical --- as an upgrading from real numbers and functions, to complex numbers and (operator) matrix algebra. The central object becomes not a set of coordinates, but a complex function of those coordinates, the wavefunction. Analogously, in gauge theory, the basic building block is typically a vector (or tensor) field. The curvature in this vectorial space is nothing but a field strength, i.e. a generalised electric and/or magnetic field. It is not difficult to see that studying differential geometry and topology one gets insights on electromagnetic-like phenomena. When the configurations of these fields and spaces are non-trivial, exotic physics arise in the form of fractal energy spectra, anyons, unconventional phase transitions and dualities. These are the cases we will deal with and from which we will recover well-known concepts.

Format:

Given the relevance, yet fuzziness, of gauge theory in physics and mathematics, I propose starting a program that allows participants to get familiar with some of these concepts. This should stimulate some scientific discussions and hopefully be also applicable to their research.

This mini course will be a survey of topics that are implicit in today's physical understanding of quantum mechanics and field theory, condensed matter, quantum optics, quantum information, and quantum simulation & computing. The topics chosen are believed to be relevant for context in the field of quantum technologies, as well as beautiful and practically useful. I propose to give an overview on stuff that might sound exotic, but that I believe to be underlying a lot of the current work that is being done in Quantum Error Correction (QEC), with ramifications in many other subdisciplines.

In order to stablish a concrete format for the sessions, I propose:

- 2h sessions (1h Lecture + 1h Discussion) every 15 days.
- **OR:** Alternating 1h lecture and 1h Discussion every week. I.e. Lecture (week 1) – Discussion (week 2).

Hours:

- For each hour of lecture, we will aim to cover a block of the syllabus.
- For each hour of discussion, we will discuss a selected topic that the participants found relevant.

Roughly 2h of home study for every lecture. 1h for every discussion.

This is a total of 10h a month! which I don't feel is too much.

Minimal Requirements:

I will assume:

- Basic knowledge of analytical mechanics and classical field theory.
- Basic knowledge of electromagnetism and light-matter interaction.
- Familiarity with Einstein's index notation, a.k.a. tensor or relativistic notation.
- Intermediate quantum mechanics: First, second quantisation, and canonical formalism.

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Scope/Syllabus:

- **Session 0:** Getting Familiar with Fields, Symmetries and Gauges
    - Crash Course in Mechanics
    - Index Notation
    - What is a Field
    - The Gauge Field
    - Canonical Structure of the Schrödinger Field

- **Session 1:** Electromagnetism as a Gauge Theory
    - Maxwell's Action
    - Maxwell's Equations
    - Canonical Structure of the (Maxwell) Gauge Field
    - Minimal Coupling
    - Matter as Charges
    - Quantum Matter-Gauge Systems
    - *Bonus*: Quantisation of Electromagnetism*

- **Session 2:** Topologically Non-Trivial Gauge Fields
    - Crash Course on Topology in Physics
    - The Berry Phase
    - The Aharonov-Bohm Effect
    - Introduction to Chern-Simons theory
    - Quantum Geometry
    - *Bonus*: Gauge Defects*

- **Session 3:** Anyon Physics
    - Chern-Simons Theory as an alternative Electromagnetism
    - Chern-Simons Theory as a many-body Aharonov-Bohm
    - Chern-Simons Theory as a Topological Field Theory
    - Anyons, Flux-tubes and Statistical transmutation
    - Anyons on the Lattice. Kitaev Physics
    - Anyon Zoo, Majoranas and Isings. Brading, Fusing and Condensing
    - *Bonus*: Fractionalisation and FQHE*

- **Session 4:** Less Conventional Gauge Stuff
    - Electric-Magnetic Duality (101)
    - Dirac's and 't Hooft-Polyakov's Magnetic Monopoles
    - Theta Term, Witten Effect and Schwinger Dyons
    - Non-Abelian Electromagnetism. Yang-Mills
    - Higher Gauge Theory. p-form Electrodynamics
    - BF Terms and More…

- **Session 5:** Phases of Gauge Theories on Lattices
    - Background Gauge Fields on the Lattice. Hofstadter's Butterfly
    - Continuum vs Lattice Gauge Theory. Wilson and 't Hooft Loops
    - Elitzur's Theorem
    - No Matter. Confinement – Deconfinement
    - Matter + Gauge. Full Phase Diagram
    - Introduction to Spin Liquids and Ices
    - Topological Order, Entanglement Entropy and Quantum Information

- **Session 6:** Synthetic and Emergent Fields
    - Spins and Wess-Zumino-Witten
    - Emergent Gauge Fields in Spin Models
    - Synthetic Gauge Fields in Continuum
    - Gauge Theory of Elasticity
    - Fractons

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- **Session 7:** Gauge and Dualities in Physics and Codes
    - Gauge – Code Correspondence
    - S-duality. Electric-Magnetic, Particle-Soliton, Kramers-Wannier
    - CSS is E+M
    - Color Codes and Subsystem Codes
    - Tanner Codes
    - *Bonus*: Geometric Langlands Program*

- **Session 8:** Mathematics of Gauge Theories and Other
    - Geometry vs Topology
    - Connections on Fiber Bundles
    - From Gauss, to Gauss-Bonnet, to Chern-Gauss-Bonnet
    - Gauge Anomalies
    - Pseudo-Riemannian geometry and gravity
    - AdS/CFT and Strings

Suggested Discussion Topics:

Up to the attendees.

References:

I have not been able to find a good complete set of notes, but the closest thing I have seen are either David Tong's notes [**1**] on Gauge Theory (although only part of it is relevant) and Eduardo Fradkin's lectures that evolved into the textbook in Ref. [**2**]. I aim to provide some notes on my own without any expectation for them to be complete or rigorous.

[**1**] D. Tong's Notes on Gauge Theory (https://www.damtp.cam.ac.uk/user/tong/gaugetheory.html).
[**2**] E. Fradkin, "Quantum Field Theory. An integrated Approach", *Princeton University Press* (2021).
Lectures available for free at (http://eduardo.physics.illinois.edu/phys582/physics582.html and https://eduardo.physics.illinois.edu/phys583/physics583.html).
[**3**] H. Nastase, "Classical field theory", *Cambridge University Press* (2019).
[**4**] M. Nakahara, "Geometry, Topology and Physics", *CRC Press* (2003).
[**5**] R. Moessner and J.E. Moore, "Topological phases of matter", *Cambridge University Press* (2021).
[**6**] J. B. Kogut, "An introduction to lattice gauge theory and spin systems", *Rev. Mod. Phys.* ***51***, 659 (1979).
[**7**] A. M. Polyakov, "Gauge Fields and Strings", *Harwood* (1987).
[**8**] S. Coleman, "Aspects of Symmetry", *Cambridge University Press* (1985).
[**9**] David Tong's Notes on the Quantum Hall Effect (https://www.damtp.cam.ac.uk/user/tong/qhe.html).
[**10**] Terry Tao's blog post on gauge theory (https://terrytao.wordpress.com/2008/09/27/what-is-a-gauge/).

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